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An Iterative Technique to Correct Probe Position Errors in Planar Near-Field to Far-Field Transformations

by

Lorant A. Muth and Richard L. Lewis

We have developed a general theoretical procedure to take into account probe position errors when planar near-field data are transformed to the far field. If the probe position errors are known, we can represent the measured data as a Taylor series, whose terms contain the error function and the ideal spectrum of the antenna. Then we can solve for the ideal spectrum in terms of the measured data and the measured position errors by inverting the Taylor series. This is complicated by the fact that the derivatives of the ideal data are unknown; that is, they can only be approximated by the derivatives of the measured data. This introduces additional computational errors, which must be properly taken into account. We have shown that the first few terms of the inversion can be easily obtained by simple approximation techniques, where the order of the approximation is easily specified. A more general solution can also be written by formulating the problem as an integral equation and using the method of successive approximations to obtain a general solution. An important criterion that emerges from the condition of convergence of the solution to the integral equation is that the total averaged position error must be less than some fraction of the sampling criterion for the antenna under test.

Key words: error-contaminated spectrum; ideal spectrum; integral equation; inversion of Taylor series; method of successive approximations; probe position errors; Taylor series expansion

1. Introduction

In planar near-field scanning a probe antenna scans the field radiated by the antenna-under-test in a plane that is located a distance z_0 away from the test antenna. Ideally, measurements are made on a regularly space grid along the x and y directions, and at a fixed distance of separation z_0 between the antennas along the z-direction. Naturally, the ideal measurement grid can only be approximated in practice, that is, position errors in all the coordinate directions are inevitable. z position errors have the most significant effect on the far field when the main beam is along the z axis [1,2]. More generally, position errors in the direction of the main beam are most significant, because they introduce errors that are proportional to $k\delta z$, whereas displacements in the orthogonal directions introduce errors that are proportional to $1/l$, where l is the characteristic scale of the near field of the

antenna. Usually $l \gg \lambda$. Consequently, first-order corrections of phase errors in near-field data is the most significant step we can take to improve accuracy of far-field features, such as gain, sidelobe levels, cross polarization, boresight direction and position of nulls. For example, the relevant phase error parameter for millimeter wave antennas is as follows: the number of near-field points needed is very large, since the sampling theorem demands data points at $\lambda/2$ spacing for complete characterization of the far field. At 60 GHz the data spacing is 2.5 mm, and the maximum position error on our near-field range, for example, is approximately 0.2 mm, (which can be reduced to 0.025 mm, if the range is carefully realigned). Such a position error represents a maximum phase error of $.4\pi/\lambda = 14.4°$ in the main beam direction. Since the near field is a superposition of an infinite number of plane waves, the actual phase error at any point could be significantly different from the main beam contribution. We would like to be able to correct such phase errors in the near field. Furthermore, second- order phase and amplitude corrections might also be necessary to achieve high accuracy in the far field.

In the rest of this report we look at the theoretical error expressions that will be used in the computer simulation study (Section 2); then we discuss the computer simulation techniques and some preliminary results that show that the major error correction technique of inverting the Taylor series approximation of the ideal data is feasible (Section 3), and give some further suggestions for studies to improve performance assessments of millimeter wave antennas (Section 4).

2. Analytical Error Expressions

The field radiated by an antenna can be described as the superposition of an infinite number of plane waves whose wavenumbers k are constant [3]. We can write that $\vec{k} = (\vec{K}, \gamma)$, where $\vec{k} \cdot \vec{k}$=constant, $\vec{K} = (k_x, k_y)$, and $\gamma^2 = k^2 - K^2$ gives the z component of the propagation vector. The received near-field signal b_0' measured by a probe whose receiving coefficients are $S_{02}'(\vec{K})$ is

$$b_0'(x,y,z) = F' a_0 \iint \vec{T}_{10}(\vec{K}) \cdot \vec{S}_{02}'(\vec{K})\, e^{i\gamma z}\, e^{i\vec{K}\cdot\vec{P}} dk_x\, dk_y \qquad (1)$$

where $F' = 1/(1 - \Gamma_l \Gamma_\rho)$, Γ_l and Γ_ρ are reflection coefficients for the load and probe, respectively, $T_{10}(\vec{K})$ are the transmission coefficients of the antenna under test, a_0 is the amplitude of the incident wave produced by the generator at the terminal surface S_0, and z is the distance of the near-field scan plane from S_1, a plane situated in front of the antenna defining $z = 0$, and the position vector $\vec{P} = (x_1 \hat{x}, x_2 \hat{y})$, where $\hat{x}$ and $\hat{y}$ are unit vectors. Equation (1) assumes that multiple reflections are negligible; the presence of multiple reflections in a real measurement range is minimized by judiciously choosing the position of the plane of measurement and the size and design of the probe.

Since eq (1) is a Fourier transform, the quantity

$$D(\vec{K}) \equiv 4\pi^2 a_0 F' \vec{T}_{10}(\vec{K}) \cdot \vec{S}_{02}'(\vec{K}) \qquad (2)$$

can be immediately written in terms of the near field. Thus,

$$D(\vec{K}) = \iint b_0'(\vec{P}, z) e^{-i\vec{k}\cdot\vec{x}} dx\, dy, \tag{3}$$

where $\vec{x} = (x_1, x_2, x_3) = \vec{P} + x_3\hat{z}$.

Since the z dependence of the near-field quantity b_0' in eq (1) appears only in the exponential, we can immediately write that

$$\frac{\partial b_0'}{\partial z} = \frac{i}{4\pi^2} \iint D(\vec{K})\, \gamma\, e^{i\gamma z} e^{i\vec{K}\cdot\vec{P}}\, dk_x\, dk_y \tag{4}$$

and

$$\frac{\partial^n b_0'}{\partial z^n} = \frac{i^n}{4\pi^2} \iint D(\vec{K})\, \gamma^n\, e^{i\gamma z} e^{i\vec{K}\cdot\vec{P}}\, dk_x\, dk_y. \tag{5}$$

Similarly, the partial derivatives with respect to x_j for $j = 1, 2$ are given by

$$\frac{\partial^n b_0'}{\partial x_j^n} = \frac{i^n}{4\pi^2} \iint D(\vec{K})\, k_j^n\, e^{i\gamma z} e^{i\vec{K}\cdot\vec{P}}\, dk_x\, dk_y. \tag{6}$$

If these expressions can be evaluated, then first- order corrections can be introduced into the data. We assume that the probe's position is known accurately and is given by

$$\vec{x} + \delta\vec{x}(\vec{x}), \tag{7}$$

where $\vec{x}$ is the position of the probe on an ideal near-field range, where measurements are made on a regularly spaced (x_1, x_2) grid, and $\delta\vec{x}(\vec{x})$ is the deviation in the probe's position from the ideal grid. A thorough discussion of the effects of such displacement errors on the far-field pattern has been presented in [2]. Some of the basic considerations relevant to the current subject are included here.

The near-field quantity $b_0'(\vec{x} + \delta\vec{x}(\vec{x}))$ is measured at the locations given by eq (7). However, this function is assumed to be defined on the regular grid $\vec{x}$ when the spectrum is obtained numerically using Fourier techniques. We can write the Taylor expansion at $\vec{x}$,

$$b_0'(\vec{x} + \delta\vec{x}(\vec{x})) \equiv b_0'(\vec{x}) + \frac{\partial b_0'(\vec{x})}{\partial x_j}\delta x_j + \frac{1}{2}\frac{\partial^2 b_0'(\vec{x})}{\partial x_i \partial x_j}\delta x_i \delta x_j + \cdots, \tag{8}$$

which defines the measured data on the left in terms of the unknown field quantities on the right. If we write $b_0' = a\exp(i\phi)$, where a is the amplitude and ϕ is the phase of b_0', then

$$\delta b_0' = e^{i\phi}\frac{\partial a}{\partial x_j}\delta x_j + iae^{i\phi}\frac{\partial \phi}{\partial x_j}\delta x_j \tag{9}$$

holds. If we assume that $a^{-1}\partial a/\partial x_j \ll \partial\phi/\partial x_j$, then eq (9) can be immediately integrated to give

$$b_0'(\vec{x} + \delta\vec{x}) = b_0'(\vec{x}) e^{i\int \kappa_j \delta x_j}, \tag{10}$$

where $\kappa_j \equiv \partial\phi/\partial x_j$ is the j component of the local wave-vector. In realistic near-field data of antennas the inequality above is satisfied for variations in the z direction. The wavenumber in the z direction can be expanded for small $\vec{K}$ as

$$k_z \equiv \gamma = k[1 - \frac{1}{2}(\frac{K}{k})^2 + \cdots] \tag{11}$$

Also, for a plane wave $\partial\phi/\partial z = k_z$. Substituting this into eq (10), we obtain, to zeroth order in (K/k), that

$$b_0'(\vec{x} + \delta\vec{x}) = b(\vec{x})e^{ik\delta z} \tag{12}$$

which is a zeroth order correction for probe displacement errors in the near field. This *plane-wave model* correction technique has been applied to real data for some time now [4]. To evaluate eq (10) more exactly, we would need to know κ_j along the path of integration, or at least at the end points of the path, since $\kappa_j \equiv \partial\phi/\partial x_j$ is an exact differential. By the mean value theorem, the integral in eq (10) can be written as $\overline{\kappa}_j \Delta x_j$, where $\overline{\kappa}_j$ is some value of κ_j in the interval of integration. Thus, the accuracy of $b_0'(\vec{x})$ will depend on the accuracy with which $\overline{\kappa}_j$ is specified.

For higher order corrections we need to develop a more thorough analysis. We can write, as in eq (3), the error-contaminated spectrum $D_e(\vec{K})$ in terms of the measured data as

$$D_e(\vec{K}) = \iint b_0'(\vec{x} + \delta\vec{x}(\vec{x}))e^{-i\vec{k}\cdot\vec{x}}\, dx\, dy. \tag{13}$$

Using expansion (8) and the transform relation (3), we can write that

$$D_e(\vec{K}) = D(\vec{K}) + \iint [\frac{\partial b_0'(\vec{x})}{\partial x_j}\delta x_j + \text{higher order terms}]e^{-i\vec{k}\cdot\vec{x}}\, dx\, dy. \tag{14}$$

Here $j = 1, 2, 3$ and the Einstein summation convention on repeated indexes is understood. In eq (14) $D(\vec{K})$ and $\partial b_0'/\partial x_j$ are unknown. We do, however, know $\partial b_0'(\vec{x} + \delta\vec{x}(\vec{x}))/\partial x_j$ through the relationships (4)-(6). We can differentiate eq (8) with respect to x_ℓ, $\ell = 1, 2, 3$, so that

$$\frac{\partial}{\partial x_\ell} b_0'(\vec{x} + \delta\vec{x}(\vec{x})) = \frac{\partial b_0'(\vec{x})}{\partial x_\ell} + \frac{\partial}{\partial x_\ell}(\frac{\partial b_0'(\vec{x})}{\partial x_j}\delta x_j) + \cdots, \tag{15}$$

which shows that eq (14) can also be written as

$$D_e(\vec{K}) = D(\vec{K}) + \iint [\frac{\partial b_0'(\vec{x} + \delta\vec{x}(\vec{x}))}{\partial x_j}\delta x_j + O(\delta x_i \delta x_j)]e^{-i\vec{k}\cdot\vec{x}}\, dx\, dy. \tag{16}$$

We now have a first-order expression that gives the true spectrum in terms of the error-contaminated spectrum and the measured near-field data. A second-order

correction can be obtained in a similar manner. Equation (15) can be differentiated again, which immediately yields that

$$\frac{\partial^2}{\partial x_i \partial x_j} b_0'(\vec{x} + \delta\vec{x}(\vec{x})) = \frac{\partial^2 b_0'(\vec{x})}{\partial x_i \partial x_j} + O(\delta x). \tag{17}$$

The second-order term in eq (8) can now be written in terms of the known quantity $b_0'(\vec{x}+\delta\vec{x}(\vec{x}))$. However, the second-order terms neglected in the first-order approximation must now be included. These are the second and third terms appearing in eq (15). After replacing the quantities $b_0'(\vec{x})$ with their first-order approximations, the second-order approximation of the known error-contaminated spectrum in terms of the true spectrum and the known displacement error function can be written as

$$D_e(\vec{K}) = D(\vec{K}) + \iint [\frac{\partial b_0'(\vec{x}+\delta\vec{x})}{\partial x_j}\delta x_j + \frac{1}{2}\frac{\partial^2 b_0'(\vec{x}+\delta\vec{x})}{\partial x_i \partial x_j}\delta x_i \delta x_j - \frac{\partial}{\partial x_\ell}(\frac{\partial b_0'(\vec{x}+\delta\vec{x})}{\partial x_j}\delta x_j)\delta x_\ell + O(\delta x_i \delta x_j \delta x_\ell)] e^{-i\vec{k}\cdot\vec{x}}\, dx\, dy. \tag{18}$$

There is a general procedure for writing the nth-order approximation to $D(\vec{K})$. Equation (8) can be rewritten as [2]

$$D_e(\vec{K}) = \int D(\vec{K}') \int e^{i(\vec{k}'-\vec{k})\cdot\vec{x}} e^{i\vec{k}'\cdot\delta\vec{x}(\vec{x})}\, d^2x\, dk_x'\, dk_y'. \tag{19}$$

This expression can be recast into the form of an inhomogeneous integral equation, which, under certain conditions, has an iterative solution [5]. Some aspects of this are detailed in Appendix A.

3. Computer Simulations

Simulation Techniques

To study the effects of probe errors using computer simulations an error-free data set is assumed in a plane of measurement, denoted by z_0. An error-free spectrum can then be obtained using standard Fourier transform techniques. An inverse Fourier transform then can yield error-free near-field data at any z_i=constant plane. If a large set of near-field data is obtained at many different z_i, then a set of error-contaminated data can be constructed on the mathematical plane z_0, according to an arbitrary error displacement function in the z direction. The error-contaminated data then yield an error-contaminated far-field pattern, which can then be compared to the original error-free spectrum. The same technique can be used to introduce displacement errors in the xy plane, which then can be studied similarly.

A number of error correction schemes can be studied with this technique since both error-free and error-contaminated data can be generated in a prescribed manner. The following error correction schemes are worthy of consideration:

1. First order phase-error corrections in the main beam direction only.
2. First order phase-error corrections taking into account the contributions of plane waves in the off-axis directions.
3. Second order phase-error correction.
4. General second order complex error corrections for x, y and z displacement errors.
5. General higher order complex error corrections for x, y and z displacement errors.

Numerical Feasibility Study

In order to execute the numerical studies suggested above, the feasibilty of the general error correction techniques has to be ascertained; that is, the convergence of the Taylor series has to be demonstrated in a simple test case. For this purpose, an actual near field of an array antenna radiating at 3.3 GHz was transformed from z_0, the original plane of measurement, to $z_1 = z_0 + 0.02\lambda$. This separation distance was chosen to scale the limits of displacement errors in a well aligned near-field range at 60 GHz, where these error correction techniques are essential to obtain acceptable far-field patterns from near-field data. The data set chosen for this study has no special significance other than it was easily available; data sets at higher frequencies were not available at the time of this study.

In Table 1, the near-field values at two different z positions separated by $\delta z = 0.02\lambda$ are listed as a function of the vertical y-direction in the center of the near-field scan plane ($x = 0$). In Table 2 the differences in amplitudes of the first, second and third order Taylor series expansions of the near-field data around z_0 (which approximate the function at z_1) and the near-field data at z_1 are shown. We can easily ascertain form these amplitude differences that the Taylor series converges to the correct value to an improved order of magnitude with each successive approximation. In Table 3 the phase differences between the phases obtained in the first, second and third order approximations and the exact phases are shown. Again, convergence in easily observed.

4. Conclusions and Suggestions for Further Study

We have developed a theoretical procedure which, in principle, can correct for probe position errors present in near-field data. The question of numerical convergence has been addressed on a simple but basic level. The result of this small study is significant in that it establishes the feasibility of obtaining corrections to error-contaminated near-field data, where the error is due to the faulty positioning of the probe. Since the feasibility of our techniques has been established a full implementation of the techniques is recommended as the next phase of this study. After successful completion of this phase, a full implementation of

the study at higher frequencies is recommended. At such higher frequencies the additional problem of an increased number of data points will have to be handled computationally; this, however, will not present fundamentally new difficulties.

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Table 1. The amplitude and phase of the near-field data at z_0 (the first two columns) and at $z_0 + 0.02\lambda$ (the third and fourth columns). $z_0 = 380$ cm and the frequency is 3.3 GHz

0.70862614E-01	-0.77406448E+02	0.71044020E-01	-0.71065971E+02
0.68881877E-01	-0.77066040E+02	0.69043353E-01	-0.70638588E+02
0.66208757E-01	-0.76955162E+02	0.66348597E-01	-0.70416214E+02
0.63052207E-01	-0.77103645E+02	0.63171484E-01	-0.70429436E+02
0.59651922E-01	-0.77551033E+02	0.59753377E-01	-0.70720047E+02
0.56263678E-01	-0.78342728E+02	0.56350138E-01	-0.71338348E+02
0.53143855E-01	-0.79519890E+02	0.53216860E-01	-0.72334320E+02
0.50533973E-01	-0.81101540E+02	0.50592951E-01	-0.73740471E+02
0.48644681E-01	-0.83061073E+02	0.48687033E-01	-0.75548035E+02
0.47639515E-01	-0.85307442E+02	0.47661930E-01	-0.77684982E+02
0.47619592E-01	-0.87686134E+02	0.47619943E-01	-0.80011765E+02
0.48612926E-01	-0.90009270E+02	0.48591554E-01	-0.82346024E+02
0.50571788E-01	-0.92103424E+02	0.50531447E-01	-0.84508301E+02
0.53379204E-01	-0.93848976E+02	0.53323638E-01	-0.86364479E+02
0.56861863E-01	-0.95192131E+02	0.56793861E-01	-0.87842964E+02
0.60805749E-01	-0.96131607E+02	0.60725752E-01	-0.88926064E+02
0.64972103E-01	-0.96695152E+02	0.64877853E-01	-0.89628761E+02
0.69113575E-01	-0.96918556E+02	0.69000728E-01	-0.89977943E+02
0.72990544E-01	-0.96832283E+02	0.72853938E-01	-0.89998314E+02
0.76387547E-01	-0.96455200E+02	0.76222844E-01	-0.89705132E+02
0.79129122E-01	-0.95793037E+02	0.78934476E-01	-0.89101845E+02
0.81094168E-01	-0.94839607E+02	0.80871642E-01	-0.88180779E+02
0.82226910E-01	-0.93579697E+02	0.81983767E-01	-0.86925827E+02
0.82544021E-01	-0.91993576E+02	0.82293473E-01	-0.85316986E+02
0.82135588E-01	-0.90063461E+02	0.81896998E-01	-0.83337311E+02
0.81159256E-01	-0.87782776E+02	0.80957592E-01	-0.80982773E+02
0.79826355E-01	-0.85168304E+02	0.79690382E-01	-0.78275276E+02
0.78379579E-01	-0.82274063E+02	0.78338675E-01	-0.75277008E+02
0.77063955E-01	-0.79202957E+02	0.77143587E-01	-0.72101707E+02
0.76095171E-01	-0.76109612E+02	0.76311313E-01	-0.68915894E+02
0.75632460E-01	-0.73187904E+02	0.75986728E-01	-0.65923607E+02
0.75764105E-01	-0.70642082E+02	0.76240897E-01	-0.63335045E+02
0.76509200E-01	-0.68648933E+02	0.77076137E-01	-0.61328175E+02
0.77832721E-01	-0.67324738E+02	0.78444391E-01	-0.60016930E+02
0.79664603E-01	-0.66707481E+02	0.80268413E-01	-0.59435833E+02
0.81913017E-01	-0.66757774E+02	0.82456067E-01	-0.59542557E+02
0.84467262E-01	-0.67374077E+02	0.84902853E-01	-0.60233273E+02
0.87192379E-01	-0.68415756E+02	0.87485127E-01	-0.61364899E+02
0.89921571E-01	-0.69726944E+02	0.90050764E-01	-0.62778072E+02
0.92453308E-01	-0.71157341E+02	0.92413932E-01	-0.64317627E+02
0.94557285E-01	-0.72575905E+02	0.94359033E-01	-0.65846565E+02

Table 2. The differences between the amplitude of the near-field data at z_1 and the amplitudes of the first, second and third-order Taylor series approximations of these near-field values

-0.45118481E-03	-0.30323863E-05	0.54389238E-06
-0.44924021E-03	-0.31217933E-05	0.54389238E-06
-0.44508278E-03	-0.31739473E-05	0.55134296E-06
-0.43936074E-03	-0.32261014E-05	0.54389238E-06
-0.43277442E-03	-0.32335520E-05	0.54761767E-06
-0.42614713E-03	-0.32037497E-05	0.55879354E-06
-0.42039528E-03	-0.31515956E-05	0.54389238E-06
-0.41628256E-03	-0.30025840E-05	0.56251884E-06
-0.41457266E-03	-0.28051436E-05	0.56996942E-06
-0.41580573E-03	-0.25555491E-05	0.56251884E-06
-0.42020157E-03	-0.22388995E-05	0.58859587E-06
-0.42781979E-03	-0.19259751E-05	0.59977174E-06
-0.43842569E-03	-0.16354024E-05	0.60349703E-06
-0.45155734E-03	-0.13858080E-05	0.61839819E-06
-0.46665967E-03	-0.11883676E-05	0.62584877E-06
-0.48295408E-03	-0.10319054E-05	0.64447522E-06
-0.49973279E-03	-0.91642141E-06	0.65565109E-06
-0.51622093E-03	-0.82701445E-06	0.66310167E-06
-0.53173304E-03	-0.75995922E-06	0.67800283E-06
-0.54571778E-03	-0.70780516E-06	0.68545341E-06
-0.55778027E-03	-0.66310167E-06	0.70035458E-06
-0.56769699E-03	-0.67055225E-06	0.71525574E-06
-0.57550520E-03	-0.73015690E-06	0.71525574E-06
-0.58130920E-03	-0.86426735E-06	0.72270632E-06
-0.58547407E-03	-0.10877848E-05	0.73015690E-06
-0.58839470E-03	-0.14156103E-05	0.74505806E-06
-0.59053302E-03	-0.18551946E-05	0.76740980E-06
-0.59235841E-03	-0.24512410E-05	0.74505806E-06
-0.59417635E-03	-0.30845404E-05	0.78231096E-06
-0.59643388E-03	-0.37848949E-05	0.78231096E-06
-0.59945136E-03	-0.44405460E-05	0.80466270E-06
-0.60362369E-03	-0.50142407E-05	0.79721212E-06
-0.60918182E-03	-0.54091215E-05	0.80466270E-06
-0.61618537E-03	-0.55581331E-05	0.81956387E-06
-0.62443316E-03	-0.54985285E-05	0.81956387E-06
-0.63331425E-03	-0.51930547E-05	0.81956387E-06
-0.64200163E-03	-0.46640635E-05	0.84191561E-06
-0.64958632E-03	-0.40382147E-05	0.81211329E-06
-0.65503269E-03	-0.33155084E-05	0.78976154E-06
-0.65743923E-03	-0.25480986E-05	0.79721212E-06
-0.65617263E-03	-0.18253922E-05	0.78231096E-06

Table 3. The differences between the near-field phase at z_1 and the phases of the first, second and third-order Taylor series approximations to these near-field values

0.47615051E-01	-0.14038086E-01	0.22888184E-03
0.49217224E-01	-0.14480591E-01	0.22888184E-03
0.51124573E-01	-0.15060425E-01	0.23651123E-03
0.53321838E-01	-0.15800476E-01	0.22125244E-03
0.55770874E-01	-0.16654968E-01	0.22125244E-03
0.58174133E-01	-0.17593384E-01	0.23651123E-03
0.60035706E-01	-0.18615723E-01	0.22888184E-03
0.60745239E-01	-0.19592285E-01	0.25939941E-03
0.59631348E-01	-0.20484924E-01	0.25939941E-03
0.56427002E-01	-0.21118164E-01	0.27465820E-03
0.51422119E-01	-0.21423340E-01	0.30517578E-03
0.45364380E-01	-0.21400452E-01	0.28991699E-03
0.39352417E-01	-0.20996094E-01	0.32806396E-03
0.33981323E-01	-0.20370483E-01	0.33569336E-03
0.29586792E-01	-0.19615173E-01	0.34332275E-03
0.26054382E-01	-0.18836975E-01	0.33569336E-03
0.23193359E-01	-0.18096924E-01	0.33569336E-03
0.20782471E-01	-0.17448425E-01	0.32806396E-03
0.18684387E-01	-0.16914368E-01	0.32806396E-03
0.16906738E-01	-0.16502380E-01	0.33569336E-03
0.15472412E-01	-0.16242981E-01	0.32806396E-03
0.14572144E-01	-0.16105652E-01	0.33569336E-03
0.14411926E-01	-0.16113281E-01	0.32043457E-03
0.15319824E-01	-0.16242981E-01	0.32043457E-03
0.17639160E-01	-0.16479492E-01	0.31280518E-03
0.21736145E-01	-0.16799927E-01	0.31280518E-03
0.27877808E-01	-0.17196655E-01	0.29754639E-03
0.36117554E-01	-0.17593384E-01	0.28991699E-03
0.46089172E-01	-0.17951965E-01	0.27465820E-03
0.56945801E-01	-0.18226624E-01	0.26702881E-03
0.67359924E-01	-0.18394470E-01	0.24414063E-03
0.75893402E-01	-0.18409729E-01	0.23651123E-03
0.81283569E-01	-0.18318176E-01	0.23651123E-03
0.82736969E-01	-0.18154144E-01	0.21743774E-03
0.80207825E-01	-0.17887115E-01	0.23651123E-03
0.74066162E-01	-0.17581940E-01	0.24032593E-03
0.65208435E-01	-0.17219543E-01	0.25939941E-03
0.54622650E-01	-0.16838074E-01	0.24795532E-03
0.43506622E-01	-0.16395569E-01	0.28228760E-03
0.32730103E-01	-0.15945435E-01	0.29754639E-03
0.23017883E-01	-0.15502930E-01	0.30517578E-03

Appendix

The True Spectrum as a Solution to an Inhomogeneous Integral Equation

Equation (19) of the main text can rewritten as

$$D(\vec{K}) = D_e(\vec{K}) + \frac{1}{4\pi^2}\iint D(\vec{K}')\iint (1 - e^{i\vec{k}'\cdot\delta\vec{x}(\vec{x})})e^{i(\vec{k}'-\vec{k})\cdot\vec{x}}\,dx\,dy\,dk'_x\,dk'_y. \quad (a1)$$

Here we have used the definition of the δ-function

$$\delta(\vec{k}' - \vec{k}) = \frac{1}{4\pi^2}\iint e^{i(\vec{k}'-\vec{k})\cdot\vec{x}}\,dx\,dy. \quad (a2)$$

Equation $(a1)$ is of the form of the Fredholm integral equation [5], which in two dimensions, is written as

$$f(x,y) = g(x,y) + \int_a^b M(x,y;x',y')f(x',y')\,dx'\,dy'. \quad (a3)$$

Symbolically this is $f = g + Mf$. If we compare eq $(a1)$ and eq $(a3)$ a straightforward identification of terms is easily done.

A solution of the general integral equation $(a3)$ can be obtained by the method of successive approximation (Neumann series) [5]. Symbolically, the nth-order solution is given by

$$f_n = g + Mf_{n-1} = (1 + M + M^2 + \ldots + M^{n-1})g. \quad (a4)$$

This solution is unique if the series in eq $(a4)$ converges uniformly [6]. The condition for convergence is that the integral operator M be bounded so that its least upper bound or norm, $\|M\|$, is less than 1. An alternate condition is that the product of the range of integration and of the maximum value of the kernel is less than 1 [6]. An estimate of the norm is given by

$$\|M\|^2 < \iint |M(x,y;x',y')|^2\,dx\,dy\,dx'\,dy', \quad (a5)$$

and the kernel is, in our example,

$$M(\vec{K},\vec{K}') = \frac{1}{4\pi^2}\iint (1 - e^{i\vec{k}'\cdot\delta\vec{x}(\vec{x})})e^{i(\vec{k}'-\vec{k})\cdot\vec{x}}\,dx\,dy. \quad (a6)$$

The maximum value of the kernel occurs at $k = k'$, where its first-order approximation can be written as

$$|M^{(1)}(\vec{K},\vec{K})| = \frac{1}{4\pi^2}\Big|\iint \vec{k}\cdot\delta\vec{x}\,dx\,dy\Big|. \quad (a7)$$

Thus,

$$|M^{(1)}(\vec{K},\vec{K})| \leq \frac{k}{4\pi^2}\Big|\iint \delta\vec{x}\,dx\,dy\Big| = \frac{kA}{4\pi^2}[\delta\vec{x}] \qquad (a8)$$

where $[\delta\vec{x}]$ is the average of the error displacement function in the scan plane. Since the range of integration is $\Delta k = 2k_i$, where $k = 2\pi/\lambda$, the second condition of convergence stated above can be written, to first order, as

$$\frac{k^3 A}{\pi^2}[\delta\vec{x}] < 1,$$

or

$$\frac{[\delta\vec{x}]}{(\lambda/2)} < \frac{1}{\pi}\frac{(\lambda/2)^2}{A}. \qquad (a9)$$

If the near field is sampled at $\lambda/2$ intervals, then the ratio on the right side above is essentially the inverse of the number of measurement points N. Then the inequality $(a9)$ becomes

$$N[\delta\vec{x}] < \frac{1}{\pi}(\lambda/2) \qquad (a10)$$

which states that the *total averaged* displacement error has to be less than a fraction of the grid spacing $\lambda/2$. This is a rather stringent condition on the size of displacement errors one can accept in a near-field measurement range, if we want to recover the true spectrum from error-contaminated near-field data. The condition essentially means that the displacement errors must be small enough so that the sampling criterion, according to the sampling theorem [7], is not violated in an average sense. Thus, only displacement-error functions that average to close to 0, or of very small magnitude, will satisfy this criterion.

If $[\delta\vec{x}] \equiv 0$ then the second order approximation of the kernel must be examined. This is

$$|M^{(2)}(\vec{K},\vec{K})| = \frac{1}{8\pi^2}\Big|\iint (\vec{k}\cdot\delta\vec{x})^2\,dx\,dy\Big|, \qquad (a11)$$

which yields an upper bound of

$$|M^{(2)}(\vec{K},\vec{K})| \leq \frac{k^2}{8\pi^2}\Big|\iint \delta\vec{x}\cdot\delta\vec{x}\,dx\,dy\Big|. \qquad (a12)$$

In deriving expression $(a12)$ we have assumed that the components of $\delta\vec{x}$ are uncorrelated, so that mixed terms average to 0. Under these conditions, the smallness of the second order kernel depends entirely on the smallness of the amplitude of $\delta\vec{x}$. Now the condition of convergence of the method of successive approximations is

$$\frac{k^4}{2\pi^2}A[\delta\vec{x}\cdot\delta\vec{x}] < 1 \qquad (a13)$$

or

$$N[\delta\vec{x}\cdot\delta\vec{x}] < \frac{2}{\pi^2}(\lambda/2)^2. \qquad (a14)$$

The interpretation of this expression is similar to the one given expression $(a10)$ above. In general, both conditions $(a10)$ and $(a14)$ have to be satisfied to guarantee convergence.

To facilitate evaluation of the integrals in eq $(a1)$, we expand the exponential term containing $\delta\vec{x}$ to second order, and write the second-order iterative solution of eq $(a1)$ as (using the abbreviations $d\vec{x} \equiv dx\,dy$ and $d\vec{K} \equiv dk_x\,dk_y$)

$$D(\vec{K}) = D_e(\vec{K}) - \frac{i}{4\pi^2}\iint \delta x_j e^{-i\vec{k}\cdot\vec{x}}\left\{\iint k'_j D_e(\vec{K}')e^{i\vec{k}'\cdot\vec{x}}d\vec{K}'\right\}d\vec{x}$$
$$+\frac{1}{8\pi^2}\iint \delta x_j \delta x_\ell e^{-i\vec{k}\cdot\vec{x}}\left\{\iint k'_j k'_\ell D_e(\vec{K}')e^{i\vec{k}'\cdot\vec{x}}d\vec{K}'\right\}d\vec{x}$$
$$-\frac{1}{16\pi^4}\iint d\vec{x}\left\{\delta x_j e^{-i\vec{k}\cdot\vec{x}}\iint d\vec{K}'\left[k'_j e^{i\vec{k}'\cdot\vec{x}}\iint d\vec{x}'\left(\delta x_\ell e^{-i\vec{k}'\cdot\vec{x}'}\iint k''_\ell D_e(\vec{K}'')e^{i\vec{k}''\cdot\vec{x}'}d\vec{K}''\right)\right]\right\}. \tag{$a15$}$$

Higher-order iterations can be readily obtained, but we will not do so here. Each iterated integral above is a Fourier transform; hence, these integrals can be evaluated with FFT codes.

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We have developed a general theoretical procedure to take into account probe position errors when planar near-field data are transformed to the far field. If the probe position errors are known, we can represent the measured data as a Taylor series, whose terms contain the error function and the ideal spectrum of the antenna. Then we can solve for the ideal spectrum in terms of the measured data and the measured position errors by inverting the Taylor series. This is complicated by the fact that the derivatives of the ideal data are unknown; that is, they can only be approximated by the derivatives of the measured data. This introduces additional computational errors, which must be properly taken into account. We have shown that the first few terms of the inversion can be easily obtained by simple approximation techniques, where the order of the approximation is easily specified. A more general solution can also be written by formulating the problem as an integral equation and using the method of successive approximations to obtain a general solution. An important criterion that emerges from the condition of convergence of the solution to the integral equation is that the total averaged position error must be less than some fraction of the sampling criterion for the antenna under test.

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error-contaminated spectrum; ideal spectrum; integral equation; inversion of Taylor series; method of successive approximations; probe position errors; Taylor series expansion

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